Fermion contribution to the static quantities of arbitrarily charged vector bosons

G. Tavares-Velasco and J. J. Toscano

Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla, Apartado Postal 1152, Puebla, Pue., México

Abstract. We present an analysis of the one-loop contribution from left- and right-handed fermions to the static electromagnetic properties of an arbitrarily charged no self-conjugate vector boson V. Particular emphasis is given to the case of a no self-conjugate neutral boson V^0. Regardless the electric charge of the V boson, a fermionic loop can induce the two CP-even form factors but only one CP-odd. As a result the corresponding electric dipole moment is directly proportional to the magnetic quadrupole moment. The CP-odd form factor might be severely suppressed since it requires the presence of both left- and right-handed fermions. The behavior of the form factors is analyzed for several scenarios of the fermion masses in the context of the decoupling theorem.

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1. Introduction

It is well known that charged particles with nonzero spin can interact with the electromagnetic field not only via the monopole but also through higher order multipoles. Though these particles do receive contributions beyond the monopole at the tree level, it is interesting to investigate those contributions arising from quantum fluctuations as they can be sensitive to new physics effects. In particular, the static electromagnetic properties of charged vector bosons are parametrized by four loop-generated electromagnetic form factors: two CP-even ones ($\Delta\kappa$ and ΔQ) and two CP-odd ones ($\Delta\widetilde{\kappa}$ and $\Delta\widetilde{Q}$). The CP-even form factors define the (anomalous) magnetic dipole and electric quadrupole moments of the vector boson, whereas the CP-odd ones parametrize its electric dipole and magnetic quadrupole moments. These form factors are model dependent and can only be generated at the one-loop level or higher orders. For instance, in the case of the charged W gauge boson of the standard model (SM), only the CP-even quantities are generated at the one-loop level [1], whereas the CP-odd ones can only be induced at higher orders [2]. Because of their sensitivity to new physics effects, the electromagnetic properties of the W boson have been the subject of considerable interest and have been studied in many extensions of the SM [3]. As far as neutral particles are concerned, by time-reversal invariance they cannot have static electromagnetic properties as long as they are characterized by self-conjugate fields, in which case the particle coincides with the antiparticle. This is true for instance for massive Majorana neutrinos [4] and the neutral gauge boson of the SM [5], which only have off-shell electromagnetic properties. The situation is different for neutral particles characterized by no self-conjugate fields, in which case the particle does not coincides with the antiparticle. For instance, massive Dirac neutrinos [6] or complex neutral vector bosons [7] do can have static electromagnetic properties. In contrast with the case of charged particles, the electromagnetic properties of neutral no self-conjugate particles can only arise from quantum fluctuations, thereby being rather sensitive to virtual effects from new particles. Even though neutral self-conjugate vector bosons are not predicted by the SM, they can arise in some of its extensions [8] and so it is interesting to study their static electromagnetic properties.

Although the $VV^\dagger\gamma$ vertex can receive contributions from fermion, boson and scalar loops, we will concentrate only on the fermion contributions. The reason for this choice is that in most of the renormalizable theories the boson or scalar particles cannot generate any CP-odd form factor at the one-loop level [9], whereas both the CP-even and CP-odd form factors do can be generated at this order by left- and right-handed fermions. This means that, the CP-odd form factors will be entirely determined by the fermion contribution in most of the renormalizable theories. In addition, while the couplings of V to gauge boson or scalar pairs would be more model-dependent, a renormalizable coupling of V to a fermion pair can be proposed by minimal substitution.

This paper has been organized as follows. In Sec. 2 the calculation of the amplitude is presented. Sec. 3 is devoted to analyze the behavior of the electromagnetic properties

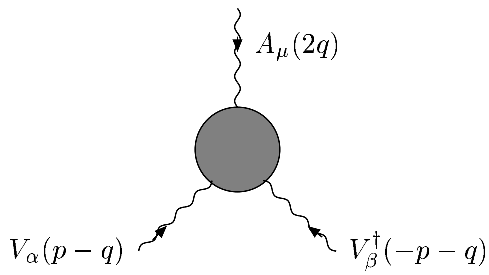

Figure 1. The $VV^\dagger\gamma$ vertex. The large dot represents the virtual effects of left- and right-handed fermions.

of the V boson in diverse scenarios of the fermion masses. Emphasis is given to the properties of the form factors in the context of the decoupling theorem [10]. The conclusions are presented in Sec. 4.

2. The on-shell $VV^\dagger\gamma$ vertex

According to the notation shown in figure 1, the most general on-shell $VV^\dagger\gamma$ vertex can be written as [7, 11]

$$\Gamma_{\alpha\beta\mu} = ie\Big\{A_V[2p_\mu g_{\alpha\beta} + 4(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu})] + 2\Delta\kappa_V(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) + \frac{4\Delta Q_V}{m_V^2}p_\mu q_\alpha q_\beta + 2\Delta\tilde{\kappa}_V\epsilon_{\alpha\beta\mu\lambda}q^\lambda + \frac{4\Delta\widetilde{Q}_V}{m_V^2}q_\beta\epsilon_{\alpha\mu\lambda\rho}p^\lambda q^\rho\Big\}. \tag{1}$$

In the case of a neutral vector boson, the A_V coefficient vanishes at any order of perturbation theory since it violates gauge invariance. The magnetic (electric) dipole moment μ_V ($\tilde{\mu}_V$) and the electric (magnetic) quadrupole moment Q_V ($\widetilde{Q}_V$) are given in terms of the electromagnetic form factors as follows

$$\mu_V = \frac{e}{2m_V}(2+\Delta\kappa_V), \tag{2}$$

$$Q_V = -\frac{e}{m_V^2}(1+\Delta\kappa_V+\Delta Q_V), \tag{3}$$

$$\tilde{\mu}_V = \frac{e}{2m_V}\Delta\tilde{\kappa}_V, \tag{4}$$

$$\widetilde{Q}_V = -\frac{e}{m_V^2}(\Delta\tilde{\kappa}_V+\Delta\widetilde{Q}_V). \tag{5}$$

As for the fermion contribution, it has been pointed out in Ref. [9] that the only fermion interaction that can contribute to the on-shell $VV^\dagger\gamma$ vertex at the one-loop level is the following fermion-gauge coupling:

$$\mathcal{L} = \bar{\psi}_1\gamma_\mu(g_L P_L + g_R P_R)\psi_2\, V^\mu + \text{H.c}, \tag{6}$$

where g_L and g_R are arbitrary complex parameters. We will present a general calculation and then discuss some specific scenarios, in particular that of a neutral no self-conjugate vector boson. The contribution of the fermion pair (ψ_1, ψ_2) to the $VV^\dagger\gamma$ vertex is given

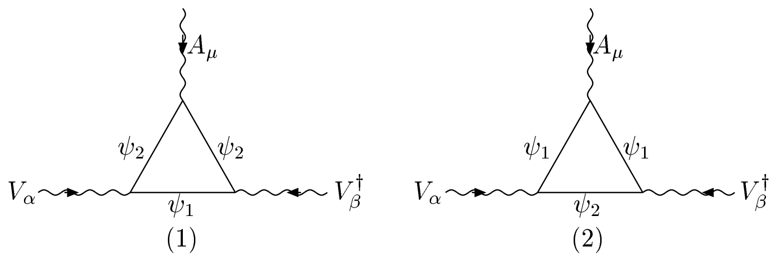

Figure 2. Feynman diagrams contributing to the $VV^\dagger\gamma$ vertex. ψ_i is a fermion with left- and right-handed couplings to the arbitrarily charged V boson.

by the two Feynman diagrams shown in figure 2. We will denote by Q_1 and Q_2 the charges of the fermions circulating in the loop. The charge of the V boson is thus $Q_V = Q_2 - Q_1$. The respective amplitude can be written down readily and the loop integral can be solved via the Feynman parameter technique. Below we will present separately the results for the CP-even and CP-odd form factors.

2.1. CP-even form factors

Once the integration over the arbitrary momentum is done, the following CP-even form factors are obtained

$$\Delta\kappa_V = \frac{Q_1}{16\pi^2}\left(\left(|g_L|^2 + |g_R|^2\right) 2\,\mathcal{I}_1 + 4\,Re(g_L\,g_R^*)\mathcal{I}_2\right) - (1 \leftrightarrow 2), \tag{7}$$

$$\Delta Q_V = -\frac{Q_1}{16\pi^2}\left(|g_L|^2 + |g_R|^2\right) 8\,\mathcal{I}_3 - (1 \leftrightarrow 2), \tag{8}$$

where $\mathcal{I}_i$ is a parametric integral depending on m_1 and m_2. We have omitted a possible color factor, which should be inserted when appropriate. $(1 \leftrightarrow 2)$ stands for an additional term in which the interchanges $Q_1 \leftrightarrow Q_2$ and $m_1 \leftrightarrow m_2$ are to be made. The parametric integrals read

$$\mathcal{I}_1 = \int_0^1 dx \int_0^{1-x} dy \left(\frac{m_1^2\,x + m_V^2 h(x,y)}{\mathcal{R}} - (1-2x-y)\log\mathcal{R}\right), \tag{9}$$

$$\mathcal{I}_2 = \int_0^1 dx \int_0^{1-x} dy \frac{m_1\,m_2(1-2x-2y)}{\mathcal{R}}, \tag{10}$$

$$\mathcal{I}_3 = \int_0^1 dx \int_0^{1-x} dy \frac{m_V^2(1-x-y)xy}{\mathcal{R}}, \tag{11}$$

with

$$\mathcal{R} = m_2^2 - (m_V^2 + m_2^2 - m_1^2)(x+y) + m_V^2(x+y)^2, \tag{12}$$

$$h(x,y) = 2y^3 + (5x-3)y^2 + (1-2x)^2 y + (x-1)x^2. \tag{13}$$

As for A_V, it is zero provided that $Q_1 = Q_2$, *i.e.* for a neutral no self-conjugate vector boson V^0. Since A_V is associated with a coupling of the vector boson to the photon, which is induced by the covariant derivative, this result reflects the fact that a neutral

particle can couple to the photon only through the field tensor $F_{\mu\nu}$. Therefore, the amplitude for the $V^0V^{0*}\gamma$ coupling is nonzero and free of ultraviolet divergences.

After some algebra, the parametric integrals can be solved explicitly:

$$\Delta\kappa_V = \frac{Q_1}{16\pi^2}\Big(\big(|g_L|^2+|g_R|^2\big)\mathcal{A}(x_1,x_2)+Re(g_Lg_R^*)\mathcal{B}(x_1,x_2)\Big)-(1\leftrightarrow 2), \tag{14}$$

$$\Delta Q_V = \frac{Q_1}{16\pi^2}\big(|g_L|^2+|g_R|^2\big)\mathcal{C}(x_1,x_2)-(1\leftrightarrow 2), \tag{15}$$

with $x_i = m_i/m_V$. The $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ functions are given by

$$\begin{aligned}\mathcal{A}(x,y) = &-\frac{1}{3}+(x^2-y^2)\big(1-2(x^2-y^2)\big)\\ &+2(x^2-y^2)\big((x^2-y^2)^2-x^2\big)\log\left(\frac{x}{y}\right)\\ &-(x^2-y^2)\big((x^2-y^2)^3+x^2(y^2-2x^2+1)+y^4\big)\frac{f(x,y)}{\delta(x,y)},\end{aligned} \tag{16}$$

$$\begin{aligned}\mathcal{B}(x,y) = 4\,x\,y\Bigg\{&2+\big(1-2(x^2-y^2)\big)\log\left(\frac{x}{y}\right)\\ &+\frac{1}{2}\big(1+2(x^2-y^2)^2-3x^2-y^2\big)\frac{f(x,y)}{\delta(x,y)}\Bigg\},\end{aligned} \tag{17}$$

$$\begin{aligned}\mathcal{C}(x,y) = \frac{2}{3}\Bigg\{&\frac{2}{3}+2(x^2-y^2)^2+y^2-3x^2\\ &-2\big((x^2-y^2)^3-2x^2(x^2-y^2)+x^2\big)\log\left(\frac{x}{y}\right)\\ &+((x^2-y^2)^4-x^2y^2-x^2(3x^4-3x^2+1)\\ &-y^2(y^4+x^2y^2-5x^4))\frac{f(x,y)}{\delta(x,y)}\Bigg\},\end{aligned} \tag{18}$$

with

$$\delta(x,y) = \sqrt{1-2(x^2+y^2)+(x^2-y^2)^2}, \tag{19}$$

$$f(x,y) = \log\left(\frac{1-(x^2+y^2)-\delta(x,y)}{1-(x^2+y^2)+\delta(x,y)}\right). \tag{20}$$

Both $\Delta\kappa_V$ and ΔQ_V are antisymmetric under the interchange $1\leftrightarrow 2$.

2.2. CP-odd form factors

The Feynman parameter technique yield

$$\Gamma^{CP-odd}_{\alpha\beta\mu} = \frac{8\,i\,e}{16\pi^2}Im(g_Lg_R^*)Q_1\mathcal{I}_4\,\epsilon_{\alpha\beta\mu\lambda}q^\lambda+(1\leftrightarrow 2), \tag{21}$$

where

$$\mathcal{I}_4 = \int_0^1 dx\int_0^{1-x}dy\frac{m_1m_2}{\mathcal{R}}. \tag{22}$$

We have used Shouten's identity to eliminate any redundant term. The form factor $\Delta\widetilde{\kappa}$ can be written as

$$\Delta\widetilde{\kappa}_V = \frac{Im(g_L g_R^*)}{16\pi^2}\, Q_1\, \mathcal{F}(x_1, x_2) + (1 \leftrightarrow 2), \tag{23}$$

where

$$\mathcal{F}(x, y) = 4\, x\, y \left(\log\left(\frac{x}{y}\right) + \left(1 - x^2 + y^2\right) \frac{f(x, y)}{2\delta(x, y)} \right). \tag{24}$$

This result is in agreement with that obtained in reference [9] for the case of the W gauge boson in the context of left-right symmetric models. From equation (23) we can see that $\Delta\widetilde{\kappa}_V$ is symmetric under the interchange $1 \leftrightarrow 2$.

Notice that there is no fermion contribution to $\Delta\widetilde{Q}$ at the one-loop level, which means that the electric dipole and magnetic quadrupole moments arising from fermion loops are entirely determined by $\Delta\widetilde{\kappa}$. This property was first noted in reference [9] for a charged W boson with left- and right-handed couplings to fermions. We have shown that this result is valid in general, regardless the charge of the gauge boson. There follows that the fermion contribution to $\widetilde{\mu}_V$ is directly proportional to $\widetilde{Q}_V$ at the one-loop level in any renormalizable theory:

$$2\widetilde{\mu}_V + m_V \widetilde{Q}_V = 0. \tag{25}$$

2.3. Form factors of a no-self conjugate neutral gauge boson

One interesting case is that of a no self-conjugate neutral gauge boson V^0. From the above expressions, one can readily obtain the results for this case once the replacement $Q_1 = Q_2 = Q$ is done:

$$\Delta\kappa_{V^0} = \frac{Q}{16\pi^2} \left(\left(|g_L|^2 + |g_R|^2\right) \mathcal{A}_0(x_1, x_2) + Re(g_L g_R^*) \mathcal{B}_0(x_1, x_2) \right), \tag{26}$$

$$\Delta Q_{V^0} = \frac{Q}{16\pi^2} \left(|g_L|^2 + |g_R|^2\right) \mathcal{C}_0(x_1, x_2), \tag{27}$$

$$\Delta\widetilde{\kappa}_{V^0} = \frac{Q\, Im(g_L\, g_R^*)}{16\pi^2} \mathcal{F}_0(x_1, x_2), \tag{28}$$

where

$$\begin{aligned}\mathcal{A}_0(x, y) &= 2(x^2 - y^2) - 2(x^2 - y^2)^2 \log\left(\frac{x}{y}\right) \\ &\quad + (x^2 - y^2) \left((x^2 - y^2)^2 - x^2 - y^2\right) \frac{f(x, y)}{\delta(x, y)},\end{aligned} \tag{29}$$

$$\mathcal{B}_0(x, y) = 8xy \left(\log\left(\frac{x}{y}\right) - (x^2 - y^2) \frac{f(x, y)}{2\delta(x, y)} \right), \tag{30}$$

$$\begin{aligned}\mathcal{C}_0(x, y) &= \frac{4}{3} \left(2(x^2 - y^2)^2 - x^2 - y^2\right) \log\left(\frac{x}{y}\right) + \frac{8}{3}(y^2 - x^2), \\ &\quad - \frac{2}{3} \left(2(x^2 - y^2)^3 + x^2(1 - 3x^2) - y^2(1 - 3y^2)\right) \frac{f(x, y)}{\delta(x, y)}\end{aligned} \tag{31}$$

$$\mathcal{F}_0(x,y) = \frac{4\,x\,y\,f(x,y)}{\delta(x,y)}. \tag{32}$$

It is clear that $\Delta\kappa_{V^0}$ and ΔQ_{V^0} are antisymmetric under the interchange $x_1 \leftrightarrow x_2$, whereas $\Delta\widetilde{\kappa}_{V^0}$ is symmetric. As a consequence the CP-even form factors of a neutral no self-conjugate boson vanish when the fermion masses are degenerate.

3. General behavior of the fermion contribution to the V form factors

3.1. Nondecoupling effects of heavy fermions

In this section we will analyze the behavior of the form factors in the decoupling limit of the fermion masses. Since $\Delta\kappa_V$ and $\Delta\widetilde{\kappa}_V$ are associated with Lorentz structures that arise from dimension-four operators, it is expected that they are sensitive to nondecoupling effects of heavy physics. On the other hand ΔQ_V and $\Delta\widetilde{Q}_V$ cannot be sensitive to this class of effects since they are associated with a Lorentz structure generated by a nonrenormalizable dimension-six operator [12]. In order to analyze the decoupling properties of the form factors of the V boson, we will focus on the loop amplitudes $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{F}$ rather than in the form factors themselves. We will thus not make any assumption about the values of the parameters g_L, g_R and the electric charges Q_1 and Q_2, which indeed are model dependent. The loop amplitudes are given in terms of the parametric integrals $\mathcal{I}_i$ as follows: $\mathcal{A}$ depends on $\mathcal{I}_1$, $\mathcal{B}$ on $\mathcal{I}_2$, $\mathcal{C}$ on $\mathcal{I}_3$, and $\mathcal{F}$ on $\mathcal{I}_4$. From the analysis of these integrals we will be able to draw some general conclusions. It is thus not necessary to consider those terms obtained after making the replacement $1 \leftrightarrow 2$.

To find whether or not any heavy physics effect is present in the loop amplitudes and found the sources of nondecoupling effects, it is convenient to analyze the behavior of the parametric integrals $\mathcal{I}_i$ in the heavy fermion limit. These integrals are given in terms of $m_V^2/\mathcal{R}$, $m_2^2/\mathcal{R}$, $m_1\,m_2/\mathcal{R}$, and $(1-2x-y)\log\mathcal{R}$. It is evident that any integral depending on $m_V^2/\mathcal{R}$ vanishes as $(m_V/m_i)^2$ when $m_i \gg m_V$, with m_i either m_1 or m_2. This is true for $\mathcal{I}_3$, which means that $\mathcal{C}$ is insensitive to the effects of heavy fermions as it only depends on this integral. Now consider the term $(1-2x-y)\log\mathcal{R}$. It can be shown that the respective integral tends to a nonzero constant value when $m_1 \to \infty$ or $m_2 \to \infty$, and vanishes when both m_1 and m_2 become large. It follows that those integrals depending logarithmically on $\mathcal{R}$ are sensitive to heavy fermions when either m_1 or m_2 is made large and the other one is kept fixed. Let us now analyze the behavior of the term $m_2^2/\mathcal{R}$, which also contribute to $\mathcal{I}_1$. When m_1 becomes large and m_2 remains small, $m_2^2/\mathcal{R} \to 0$, but in the opposite case it tends to a constant value. Of course when both m_1 and m_2 become heavy, $m_2^2/\mathcal{R}$ tend to a nonzero constant. Therefore, the integral $\mathcal{I}_1$ is sensitive to nondecoupling effects when both m_1 and m_2 are heavy or when either m_1 or m_2 is heavy. Finally, let us discuss the behavior of the term $m_1\,m_2/\mathcal{R}$, which enters into the integrals $\mathcal{I}_2$ and $\mathcal{I}_4$. When $m_1 \gg m_2$ both $\mathcal{I}_2$ and $\mathcal{I}_4$ decrease as m_2/m_1 and vice versa. On the other hand, when $m_1 \sim m_2 \gg m_V$, these integrals tend to a nonzero constant value. Therefore, $\mathcal{B}$ and $\mathcal{F}$ are sensitive to nondecoupling effects

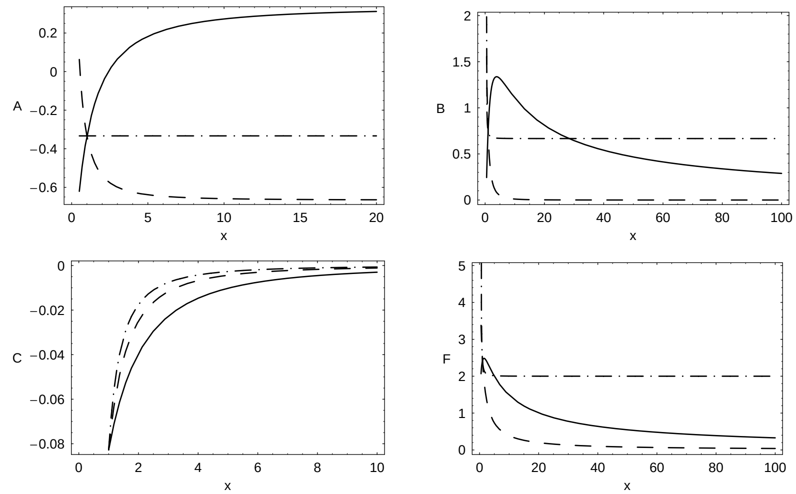

Figure 3. The fermion-loop amplitude as a function of $x_1 = m_1/m_V$ and $x_2 = m_2/m_V$. The curves corresponds to the following scenarios: $x_1 = 1$ and $x_2 = x$ (solid line), $x_2 = 1$ and $x_1 = x$ (dashed line), and $x_1 = x_2 = x$ (dot-dashed line). In order to show clearly the decoupling properties of the $\mathcal{B}$ and $\mathcal{F}$ functions, a larger range for x has been chosen because these functions decrease slowly with increasing x.

only when the fermions are degenerate, in contrast with $\mathcal{A}$, which always is sensitive to heavy fermions.

In summary, $\mathcal{C}$ is always insensitive to nondecoupling effects, $\mathcal{B}$ and $\mathcal{F}$ are sensitive to heavy fermions only when the fermion masses are degenerate, and $\mathcal{A}$ is sensitive to nondecoupling effects in both the degenerate and nondegenerate scenarios. This is illustrated graphically in figure 3. We thus can conclude that while $\Delta\kappa_V$ is sensitive to heavy fermion effects, ΔQ_V is always of decoupling nature. As far as $\Delta\widetilde{\kappa}_V$ is concerned, it is sensitive to nondecoupling effects provided that the fermion masses are degenerate. All these results are in accordance with the decoupling theorem [10], which establishes that only those Lorentz structures arising from renormalizable operators can be sensitive to nondecoupling effects, whereas those structures coming from nonrenormalizable operators are suppressed by inverse powers of the heavy mass. In most of the theories, the nondecoupling effects are unobservable since they are absorbed by renormalization [10].

It is worth analyzing separately the specific case of a neutral no self-conjugate vector boson. From equations (7) and (8) it is clear that the CP-even form factors vanish when $Q_1 = Q_2$ and $x_1 = x_2$ since they are antisymmetric under this interchange. In the case that $x_1 \neq x_2$, $\Delta\kappa_{V^0}$ is sensitive to nondecoupling effects when either $x_1 \gg x_2$ or $x_2 \gg x_1$, and the same is true for $\Delta\widetilde{\kappa}_V$. As far as ΔQ_V is concerned, it is of decoupled nature and vanishes when either x_1 or x_2 are heavy. The latter form factor goes to zero rapidly in the heavy fermion limit.

Although the actual size of these form factor is model dependent, we can infer their order of magnitude from the analysis of the loop functions. In figure 3 we show these loop functions in three scenarios of interest: $x_1 = 1$ and $x_2 = x$, $x_2 = 1$ and $x_1 = x$, and

$x_1 = x_2 = x$. The first two scenarios corresponds to the case of a fermion with mass of the order of m_V, whereas the third one corresponds to the case of degenerate fermions. From these figures we can see that $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{F}$ can be of the order of $O(1)$, whereas C is one or two orders of magnitude below. Thus if there is no large cancellations, $\Delta\kappa$ and $\Delta\widetilde{\kappa}$ can be at most of the order of $g^2/(16\pi^2)$, whereas ΔQ is one or two orders of magnitude below. However, $\Delta\widetilde{\kappa}$ requires that both g_L and g_R are nonzero and may be further suppressed.

3.2. Degeneracy and nondegeneracy of the fermion masses

Let us now analyze the static quantities of the V boson in two scenarios of interest. In particular, we will obtain explicit expressions for the form factors in the massless fermion and heavy fermion limits. In the first scenario we assume that there is degeneracy of the fermion masses and in the second case we assume $x_2 = 0$ and x_2 arbitrary, which implies that one fermion is massless or much lighter than the other one. We rewrite the static quantities of the V boson as

$$\Delta\kappa_V = \frac{1}{16\,\pi^2}\left(\left(g_L^2 + g_R^2\right)\bar{\mathcal{A}}(x) + Re(g_L\, g_R^*)\bar{\mathcal{B}}(x)\right), \tag{33}$$

$$\Delta Q_V = \frac{1}{16\,\pi^2}\left(g_L^2 + g_R^2\right)\bar{\mathcal{C}}(x), \tag{34}$$

and

$$\Delta\widetilde{\kappa}_V = \frac{1}{16\,\pi^2} Im(g_L\, g_R^*)\bar{\mathcal{F}}(x), \tag{35}$$

In the scenario with degenerate fermions ($x_1 = x_2 = x$) we obtain

$$\bar{\mathcal{A}}(x) = \frac{1}{3}(Q_2 - Q_1), \tag{36}$$

$$\bar{\mathcal{B}}(x) = 8(Q_1 - Q_2)x^2\left(1 - \sqrt{4x^2-1}\tan^{-1}\left(\frac{1}{\sqrt{4x^2-1}}\right)\right), \tag{37}$$

$$\bar{\mathcal{C}}(x) = \frac{4}{9}(Q_2 - Q_1)\left(-1 + 3x^2 + \frac{6x^2(1-2x^2)}{\sqrt{4x^2-1}}\tan^{-1}\left(\frac{1}{\sqrt{4x^2-1}}\right)\right), \tag{38}$$

$$\bar{\mathcal{F}} = 4(Q_1 + Q_2)\frac{x^2}{\sqrt{4x^2-1}}\tan^{-1}\left(\frac{1}{\sqrt{4x^2-1}}\right). \tag{39}$$

Some conclusions can be drawn from these expressions. First of all, $\bar{\mathcal{A}}$ is constant, which means that $\Delta\kappa_V$ always receives a contribution from a degenerate fermion doublet, regardless the value of the fermion mass. Notice also that $\bar{\mathcal{B}}$ vanishes for $x = 0$, whereas $\bar{\mathcal{C}}$ takes the value $(4/9)(Q_1 - Q_2)$. From here we can recover the known result for the contribution from massless fermions to the static quantities of the W boson in the SM [1]. In the heavy fermion limit, $\bar{\mathcal{B}}(x) = (2/3)(Q_1 - Q_2)$ and $\bar{\mathcal{C}}(x) = 0$, which is also in accordance with the previous discussion. As for the CP-odd form factor $\bar{\mathcal{F}}$, it vanishes when $x = 0$ and tends to $2(Q_1 + Q_2)$ in the heavy fermion limit. It means that $\Delta\widetilde{\kappa}$ is insensitive to massless fermions but sensitive to a heavy degenerate fermion doublet.

From equations (36)-(39) it is also clear that in this scenario there are no contributions to the CP-even form factors of a neutral V^0 boson, whereas the CP-odd contribution is nonzero. This result is independent of the fermion mass.

Let us now consider the scenario with nondegenerate fermions ($x_1 \neq x_2$). Without losing generality we consider $x_2 = 0$ and $x_1 = x$. We obtain

$$\bar{\mathcal{A}}(x) = \frac{1}{3}(Q_2 - Q_1)(1 + 6x^4) + (Q_1 + Q_2)x^2 + 2x^4\left((Q_2 - Q_1)x^2 + Q_1\right)\log\left(\frac{|x^2 - 1|}{x^2}\right), \tag{40}$$

$$\bar{\mathcal{C}}(x) = -\frac{4}{9}(Q_2 - Q_1)(1 + 3x^4) - \frac{2}{3}(Q_2 + 3Q_1)x^2 - \frac{4}{3}x^2\left((Q_2 - Q_1)x^4 + 2Q_1x^2 - Q_1\right)\log\left(\frac{|x^2 - 1|}{x^2}\right), \tag{41}$$

and $\bar{\mathcal{B}} = 0$. In the heavy-mass limit, $\bar{\mathcal{A}} \to -(2\,Q_1 + Q_2)/3$ and $\bar{\mathcal{C}} \to 0$, which again is in agreement with the previous discussion. As long as the CP-odd amplitude is concerned, there is no contribution to $\Delta\tilde{\kappa}_V$ in this scenario as it vanishes when either $x_1 = 0$ or $x_2 = 0$.

The respective expressions for a neutral vector boson can be readily obtained after the replacement $Q_1 = Q_2 = Q$ is done:

$$\bar{\mathcal{A}}_0(x) = 2Qx^2\left(1 + x^2\log\left(\frac{|x^2 - 1|}{x^2}\right)\right), \tag{42}$$

$$\bar{\mathcal{C}}_0(x) = -\frac{4}{3}Qx^2\left(2 + (2x^2 - 1)\log\left(\frac{|x^2 - 1|}{x^2}\right)\right). \tag{43}$$

In the heavy-mass limit, $\bar{\mathcal{A}}_0 \to -Q$ and $\bar{\mathcal{C}}_0 \to 0$. Of course when $x = 0$, the degenerate fermion case is recovered and there is no contributions to the CP-even form factors.

4. Summary

We presented a comprehensive study of the one-loop fermion contributions to the static electromagnetic properties of a no self-conjugate vector boson with arbitrary electric charge. A renormalizable coupling of the V boson to left- and right-handed fermions was assumed. Analytical expressions for the respective form factors were presented in the general case and some scenarios of interest, which can be used to evaluate the fermion contribution in the context of any renormalizable theory predicting this class of interactions. Apart from the two CP-even form factors, the fermion loop gives rise to only one CP-odd form factor, namely $\Delta\tilde{\kappa}_V$. As a consequence, the contributions to the electric dipole and magnetic quadrupole moments turn out to be directly proportional to each other. This result is valid for any no self-conjugate vector boson, regardless its electric charge. As for the CP-odd form factor $\Delta\tilde{\kappa}_V$, it turns out to be proportional to $Im(g_L\,g_R^*)$, thereby requiring the presence of both left- and right-handed fermions.

The behavior of the electromagnetic form factors of the V boson was analyzed in several scenarios. Particular emphasis was given to the heavy fermion limit, and

the consistency with the decoupling theorem was discussed. As for the CP-even form factors, it was shown that $\Delta\kappa_V$ is sensitive to heavy fermions, whereas ΔQ_V is always of decoupling nature. The latter decreases rapidly, as m_i^{-2}, when the heavier fermion mass m_i increases, which means that it is only sensitive to physics effects which are not very far from the m_V scale. The CP-odd quantity $\Delta\widetilde{\kappa}_V$ is sensitive to heavy fermions provided that the fermions are degenerate, whereas in a nondegenerate scenario it decreases as m_i^{-1}. It was also found that for fermion masses near the m_V scale, the loop amplitudes associated with $\Delta\kappa_V$ and $\Delta\widetilde{\kappa}_V$ are of the same order of magnitude, but the one associated with ΔQ_V is one order of magnitude below. Since the $\Delta\widetilde{\kappa}_V$ form factor requires the presence of both left-handed and right-handed fermions, it depends strongly on the existence of a complex phase, which is expected to be very small in most of the renormalizable theories.

We would like to emphasize that all the above remarks are valid regardless the electric charge of the V boson. An interesting case is that of a neutral no self-conjugate vector boson, for which explicit analytical expressions were obtained. In particular, it was found that there is no contribution to the CP-even form factor of the neutral vector boson when the fermion masses are degenerate, in contrast to the CP-odd form factor, which is nonzero in this scenario.

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